Color Entanglement Effects at RHIC

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Based on: **Phys.Rev. D90 (2014) 3, 034016. A. Schäfer and ZJ**
Phys.Rev. D90 (2014) 9, 094012. A. Schäfer and ZJ
Phys.Rev. D92 (2015) 1, 014034. ZJ

Outline:

- SSA in direct photon production in pA collisions
- SSA in virtual photon production(DY) in pA collisions
- SSA in photon-jet production in pA collisions
- Summary

Recent studies on SSAs in pA

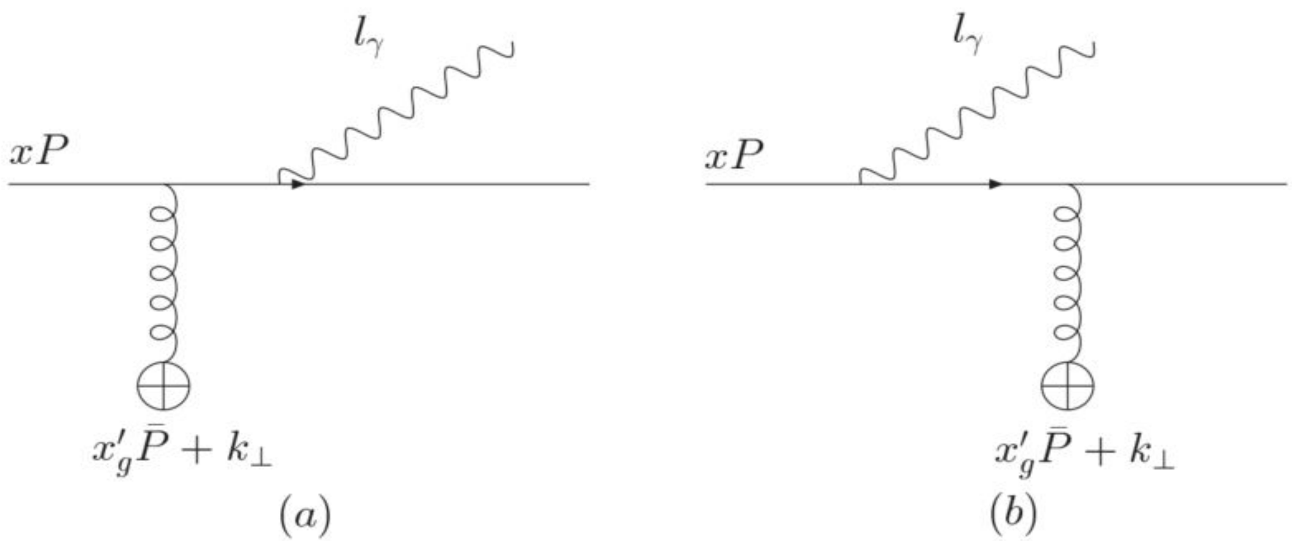
Formulated in different hybrid approaches:

Schems	Proton	Nucleus	Fragmentation
D. Boer, A. Dumitru and A. Hayashigaki, 2006	The Sivers TMD	CGC	
F. Yuan and Z. B. Kang, 2011	Twist-2 Collinear $h_1(x)$	CGC	The Collins TMD
Y. Kovchegov and M. Sievert, 2012	Twist-2 Collinear $h_1(x)$	CGC (odderon)	
Z. B. Kang and B. W. Xiao, 2013	The Sivers TMD	CGC	
A. Schaefer and JZ (2014-2015)	Twist-3 Collinear $T_F(x,x)$	CGC	

Disclaimer: Not a complete list.

Focus on the **forward region** of the incoming polarized proton.

Dominant partonic process

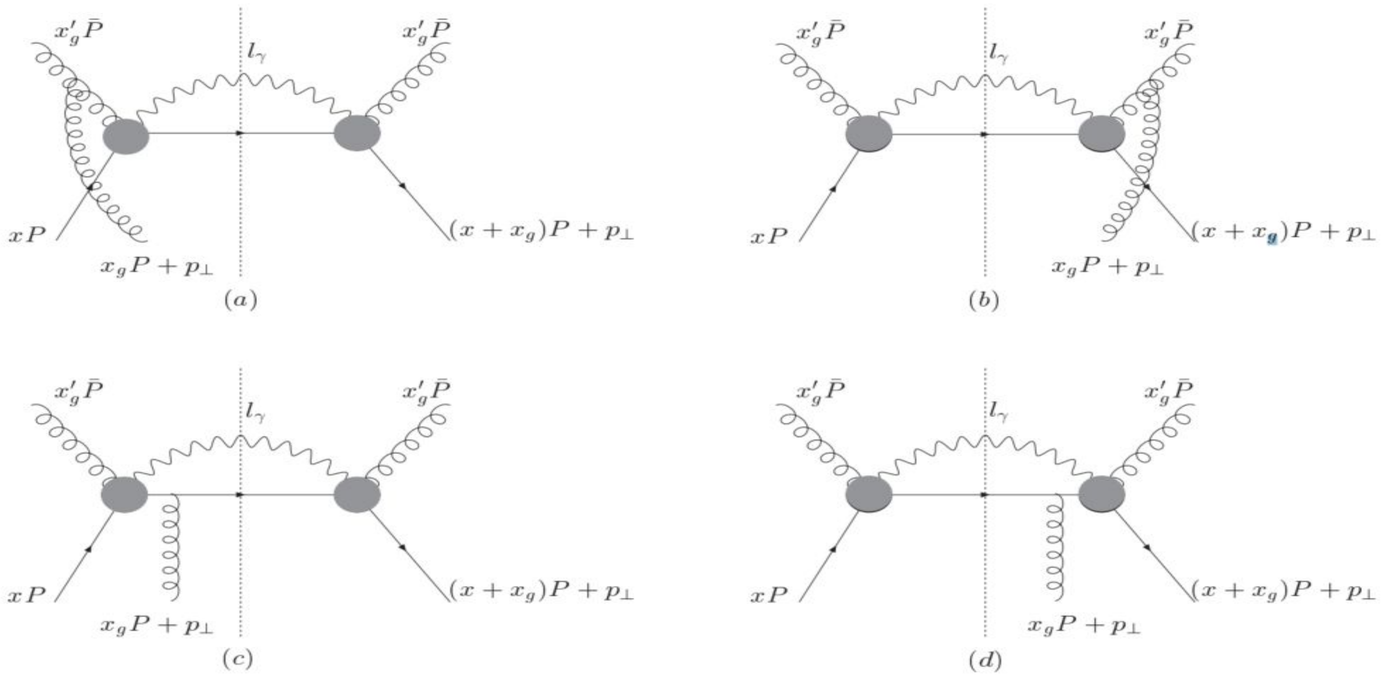


There is also interesting physics in the **backward region**, See Boer's talk.

- SSA for photon production in **polarized** pp collisions has been computed.

- Photon production in **unpolarized** pA collisions has been computed

SSA in polarized pp collinsions



polarized proton	unpolarized proton
Twist-3 collinear $T_F(x,x)$	Twist-2 Collinear $f^g(x)$

Qiu, Sterman

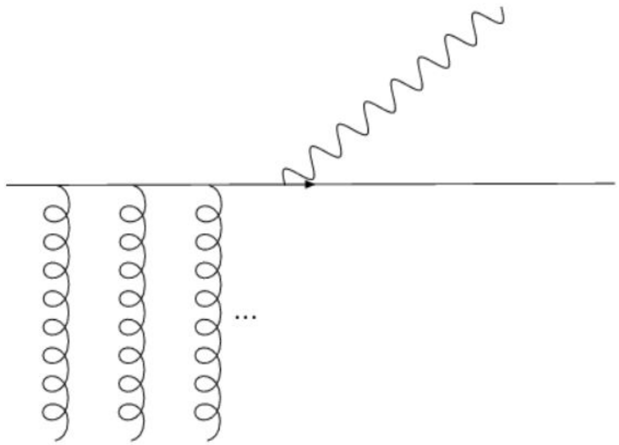
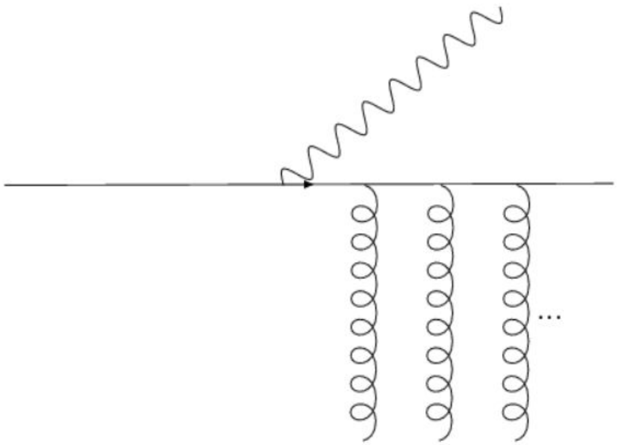
Kouvaris, Qiu, Vogelsang, Yuan

Kanazawa, Koike, Metz, Pitonyak

Twist-3 approach has been extensively used to study SSAs in pp/ep collisions.

See Metz's talk.

Photon production in unpolarized pA collisoins

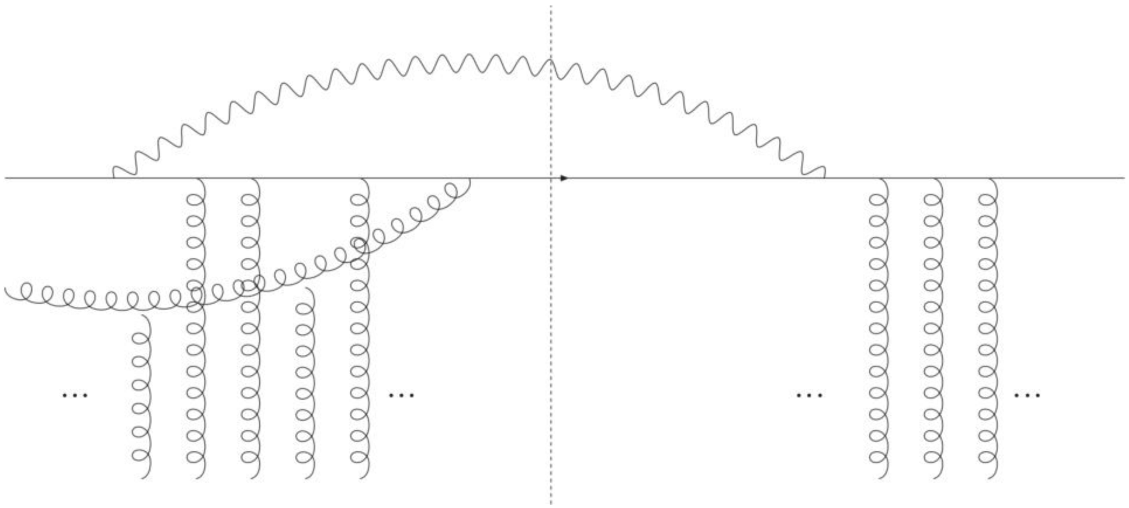
 

unpolarized proton	**unpolarized nucleus**
Twist-2 collinear $f_1(x)$	CGC $U(v_T)U^\dagger(0)$

F. Gelis and J. Jalilian-Marian

Collinear twist-3 and CGC

Generic diagram contributing to the SSA for photon production in pA:

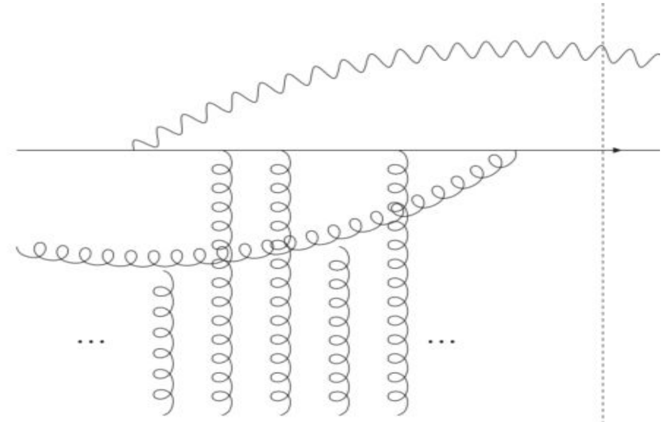


polarized proton	unpolarized nucleus
Collinear twist-3: $T_F(x,x)$	CGC: multiple point function

A. Schaefer and ZJ

Picking up the imaginary phase,

$$\frac{1}{x_g + i\epsilon} = P\frac{1}{x_g} - i\pi\delta(x_g)$$



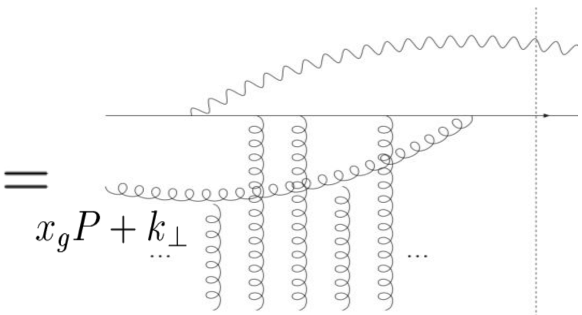
The soft gluon pole contribution,

$$\begin{aligned}
\mathcal{M}_{SGP} = & -ieg^2 \int \frac{d^2p_\perp}{(2\pi)^2}\frac{\rho_{p,a}(p_\perp)}{p_\perp^2}\int \frac{dk_1^- d^2k_{1\perp}}{(2\pi)^3}\,\bar{u}(l_q) \\
& \times \Big\{ \not{\epsilon} S_F(x_1P+q)\frac{\not{C}_U(q-k_1,p_\perp)}{(q-k_1)^2+i\epsilon} t^b S_F(x_1P+k_1)\not{n}U(k_{1\perp}) \\
& \quad + \frac{\not{C}_U(q-k_1,p_\perp)}{(q-k_1)^2+i\epsilon} t^b S_F(x_1P-l_{\gamma^*}+k_1)\not{n}U(k_{1\perp})S_F(x_1P-l_{\gamma^*})\not{\epsilon} \\
& \quad + \frac{\not{C}_U(q-k_1,p_\perp)}{(q-k_1)^2+i\epsilon} t^b S_F(x_1P-l_{\gamma^*}+k_1)\not{\epsilon} S_F(x_1P+k_1)\not{n}U(k_{1\perp}) \Big\} \\
& \times u(x_1P)\left[\tilde{U}(k_\perp-k_{1\perp}) - (2\pi)^2\delta(k_\perp-k_{1\perp})\right]_{ba} \\
& + ieg^2 \int \frac{d^2p_\perp}{(2\pi)^2}\frac{\rho_{p,a}(p_\perp)}{p_\perp^2}\int d^2x_\perp e^{ik_\perp\cdot x_\perp} \\
& \times \bar{u}(l_q)\frac{\not{n}S_F(x_1P-l_{\gamma^*})\not{\epsilon} + \not{\epsilon}S_F(x_1P+q)\not{n}}{x_gP+i\epsilon} t^b U(x_\perp)u(x_1P)\left[\tilde{U}(x_\perp)-1\right]_{ba}
\end{aligned}$$

K_T expansion

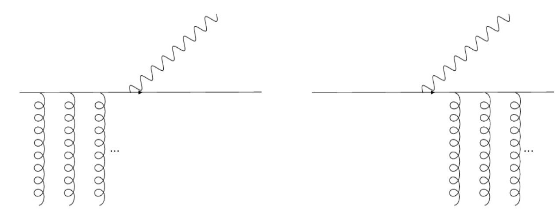
$$H(x_g P + k_\perp) = H(x_g p) + \frac{\partial H(x_g P + k_\perp)}{\partial k_\perp^i}\Big|_{k_\perp = 0} k_\perp^i + \ldots$$

$$\mathcal{M}_{SGP} =$$



$$H(x_g P + k_\perp) \propto \mathcal{M}_{SGP} \times \mathcal{M}^*$$

$$\mathcal{M}^* =$$



Derivative term contribution to the polarized cross section

$$\frac{d^3\Delta\sigma}{d^2 l_{\gamma\perp} dz} = \frac{\alpha_s \alpha_{em} N_c}{N_c^2-1} \frac{1+(1-z)^2}{z l_{\gamma\perp}^2}(z-1)\int_{x_{min}}^1 \frac{dx}{x}\int d^2k_\perp \frac{\left[\epsilon^{l_\gamma S_\perp np}-\epsilon^{k_\perp S_\perp np}\right]}{(k_\perp - l_{\gamma\perp}/z)^2 (k_\perp - l_{\gamma\perp})^2}$$
$$\times \sum_q e_q^2 \left[-x\frac{d}{dx}T_{F,q}(x,x)\right]\left[x'_g G_{DP}(x'_g,k_\perp) - x'_g G_4(x'_g,k_\perp)\right]$$

Dipole gluon TMD: $G_{DP} \propto \langle \mathrm{Tr}_c[U(x_\perp)U^\dagger(y_\perp)]\rangle$

New gluon TMD: $G_4 \propto \langle \mathrm{Tr}_c[U(x_\perp)]\mathrm{Tr}_c[U^\dagger(y_\perp)]\rangle$

In the MV model $$G_4 = \frac{1}{N_C^2}G_{DP}$$

Extrapolating to the kinematical limit $l_{\gamma\perp} \gg Q_{sq} \sim k_\perp$

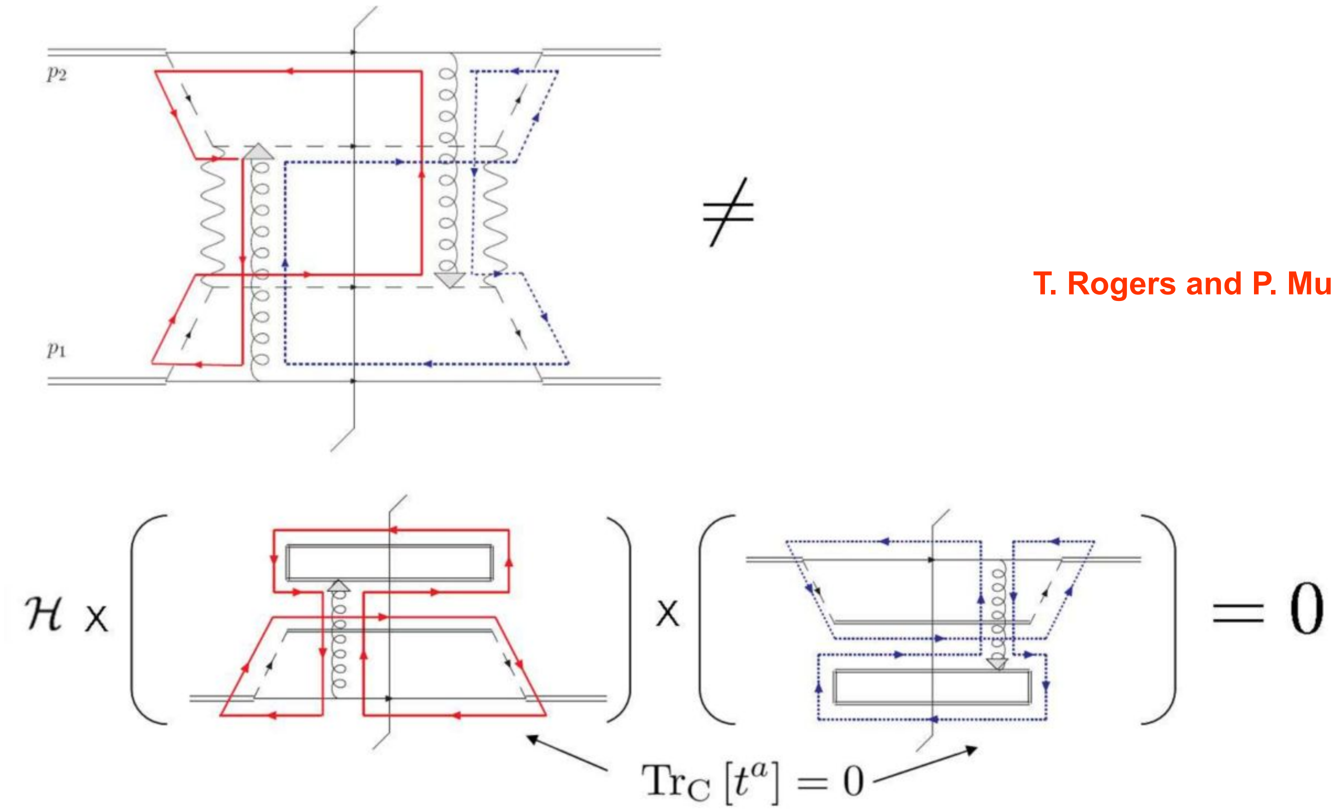
$\mathrm{Tr}_C\,[t^a] = 0$ **!**

$$\frac{d^3\Delta\sigma}{d^2 l_{\gamma\perp} dz} = \frac{\alpha_s \alpha_{em} N_c}{N_c^2-1}\epsilon^{l_\gamma S_\perp np}\frac{z[1+(1-z)^2]}{l_{\gamma\perp}^6}(z-1)$$
$$\times \sum_q e_q^2 \int_{x_{min}}^1 \frac{dx}{x}\left[-x\frac{d}{dx}T_{F,q}(x,x)\right]\left\{x'_g G(x'_g) - \frac{1}{N_c^2}x'_g G(x'_g)\right\}$$

Color entanglement effect

Collinear approach

Color Entanglement(CE) Effect



T. Rogers and P. Mulders

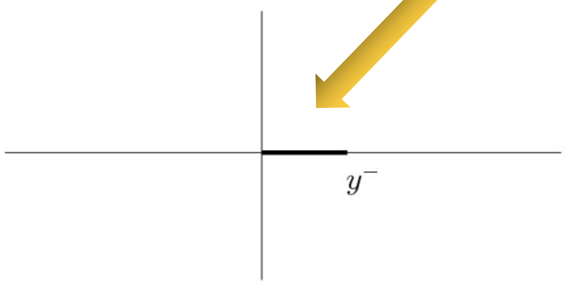
The Ward Identity argument fails.

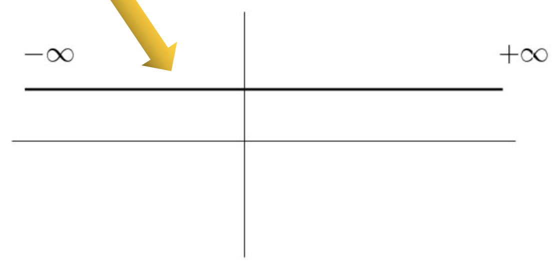
Why the WI argument fails

To decouple the longitudinal gluon, rely on $P^{\mu}\mathcal{M}_{\mu} = 0$

But the WI is: $x_g P^{\mu}\mathcal{M}_{\mu} = 0$

Gluon pole $$\frac{1}{x_g + i\epsilon} = P\frac{1}{x_g} - i\pi\delta(x_g)$$





$x_g P^{\mu}\mathcal{M}_{\mu} = 0$ When

$x_g \neq 0$	$P^{\mu}\mathcal{M}_{\mu} = 0$
$x_g = 0$	$P^{\mu}\mathcal{M}_{\mu} \stackrel{?}{=} 0$

Collinear twist-2 & CGC: CE effect is absent

Collinear twist-3 & CGC:

one additional gluon attachment from proton (not a purely longitudinal one) → CE effect is **present.**

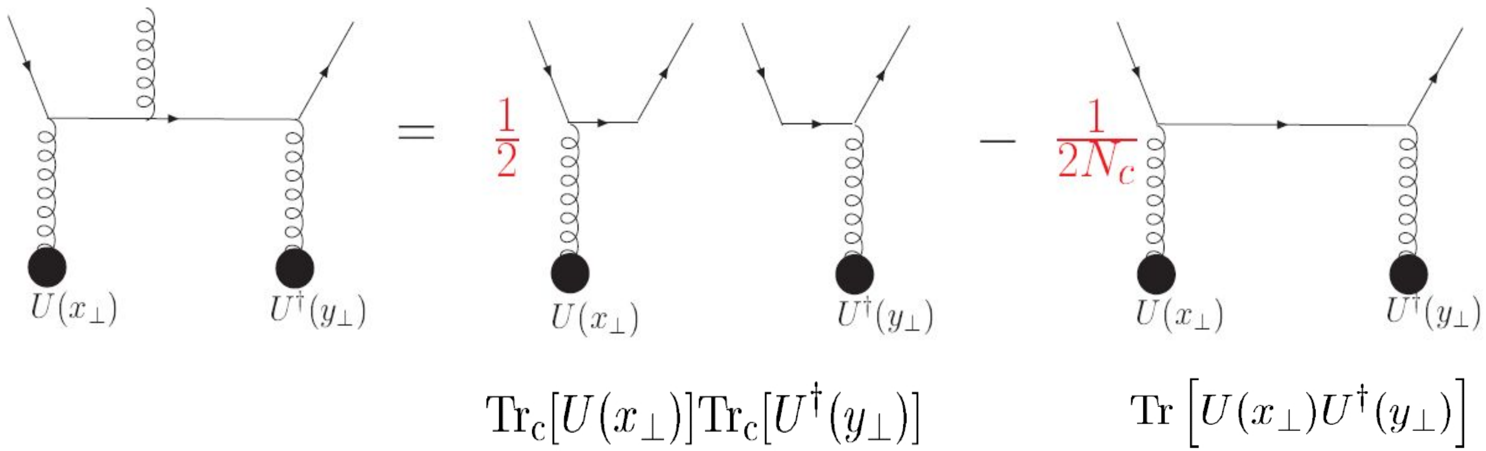
$$H(x_gP+k_\perp)=H(x_gp)+\frac{\partial H(x_gP+k_\perp)}{\partial k_\perp^i}|_{k_\perp=0}k_\perp^i+\ldots$$

all longitudinal gluon attachment from proton can be decoupled using the WI argument → CE effect doesn't jeopardize higher-twist collinear factorization

- Only a short gauge link appears in $T_F(x,x)$.

How does G_4 emerge?

A simple illustration:



coherent gluon rescattering & non abelian feature of QCD

◆**Caution:** **the nontrivial color structure appears in the case for which the Ward identity argument does not apply.**

SSA in DY in pA collisions

Hard gluon pole appears due to the existance of the additional hard scale Q

Soft gluon pole(SGP):

$$\mathcal{M}_{SGP} = -ieg^2\int\frac{d^2p_\perp}{(2\pi)^2}\frac{\rho_{p,a}(p_\perp)}{p_\perp^2}\int\frac{dk_1^- d^2k_{1\perp}}{(2\pi)^3}\,\bar{u}(l_q)$$
$$\times\Big\{\not\epsilon S_F(x_1P+q)\frac{\not C_U(q-k_1,p_\perp)}{(q-k_1)^2+i\epsilon}t^b S_F(x_1P+k_1)\not n U(k_{1\perp})$$
$$+\frac{\not C_U(q-k_1,p_\perp)}{(q-k_1)^2+i\epsilon}t^b S_F(x_1P-l_{\gamma^*}+k_1)\not n U(k_{1\perp})S_F(x_1P-l_{\gamma^*})\not\epsilon$$
$$+\frac{\not C_U(q-k_1,p_\perp)}{(q-k_1)^2+i\epsilon}t^b S_F(x_1P-l_{\gamma^*}+k_1)\not\epsilon S_F(x_1P+k_1)\not n U(k_{1\perp})\Big\}$$
$$\times u(x_1P)\left[\tilde{U}(k_\perp-k_{1\perp})-(2\pi)^2\delta(k_\perp-k_{1\perp})\right]_{ba}$$
$$+\ ieg^2\int\frac{d^2p_\perp}{(2\pi)^2}\frac{\rho_{p,a}(p_\perp)}{p_\perp^2}\int d^2x_\perp e^{ik_\perp\cdot x_\perp}$$
$$\times\bar{u}(l_q)\frac{\not n S_F(x_1P-l_{\gamma^*})\not\epsilon+\not\epsilon S_F(x_1P+q)\not n}{x_gP+i\epsilon}t^b U(x_\perp)u(x_1P)\left[\tilde{U}(x_\perp)-1\right]_{ba}$$

Hard gluon pole(HGP):

$$\mathcal{M}_{HGP} = -ieg^2\int\frac{d^2p_\perp}{(2\pi)^2}\frac{\rho_{p,a}(p_\perp)}{p_\perp^2}\int\frac{dk_1^- d^2k_{1\perp}}{(2\pi)^3}\,\bar{u}(l_q)$$
$$\times\Big\{\frac{\not C_U(q-k_1,p_\perp)}{(q-k_1)^2+i\epsilon}t^b S_F(x_1P-l_{\gamma^*}+k_1)\not n U(k_{1\perp})S_F(x_1P-l_{\gamma^*})\not\epsilon$$
$$+\frac{\not C_U(q-k_1,p_\perp)}{(q-k_1)^2+i\epsilon}t^b S_F(x_1P-l_{\gamma^*}+k_1)\not\epsilon S_F(x_1P+k_1)\not n U(k_{1\perp})\Big\}$$
$$\times u(x_1P)\left[\tilde{U}(k_\perp-k_{1\perp})-(2\pi)^2\delta(k_\perp-k_{1\perp})\right]_{ba}$$
$$+ieg^2\int\frac{d^2p_\perp}{(2\pi)^2}\frac{\rho_{p,a}(p_\perp)}{p_\perp^2}\int d^2x_\perp e^{ik_\perp\cdot x_\perp}$$
$$\times\Big\{\bar{u}(l_q)\frac{\not n S_F(x_1P-l_{\gamma^*})\not\epsilon}{x_gP+i\epsilon}t^b U(x_\perp)u(x_1P)\left[\tilde{U}(x_\perp)-1\right]_{ba}$$
$$+i\bar{u}(l_q)\not p t^a S_F(x_1P-l_{\gamma^*}+k)\not n\left[U(x_\perp)-1\right]S_F(x_1P-l_{\gamma^*})\not\epsilon u(x_1P)$$
$$+i\bar{u}(l_q)\not n\left[U(x_\perp)-1\right]S_F(l_q-k)\not p t^a S_F(x_1P-l_{\gamma^*})\not\epsilon u(x_1P)\Big\}$$

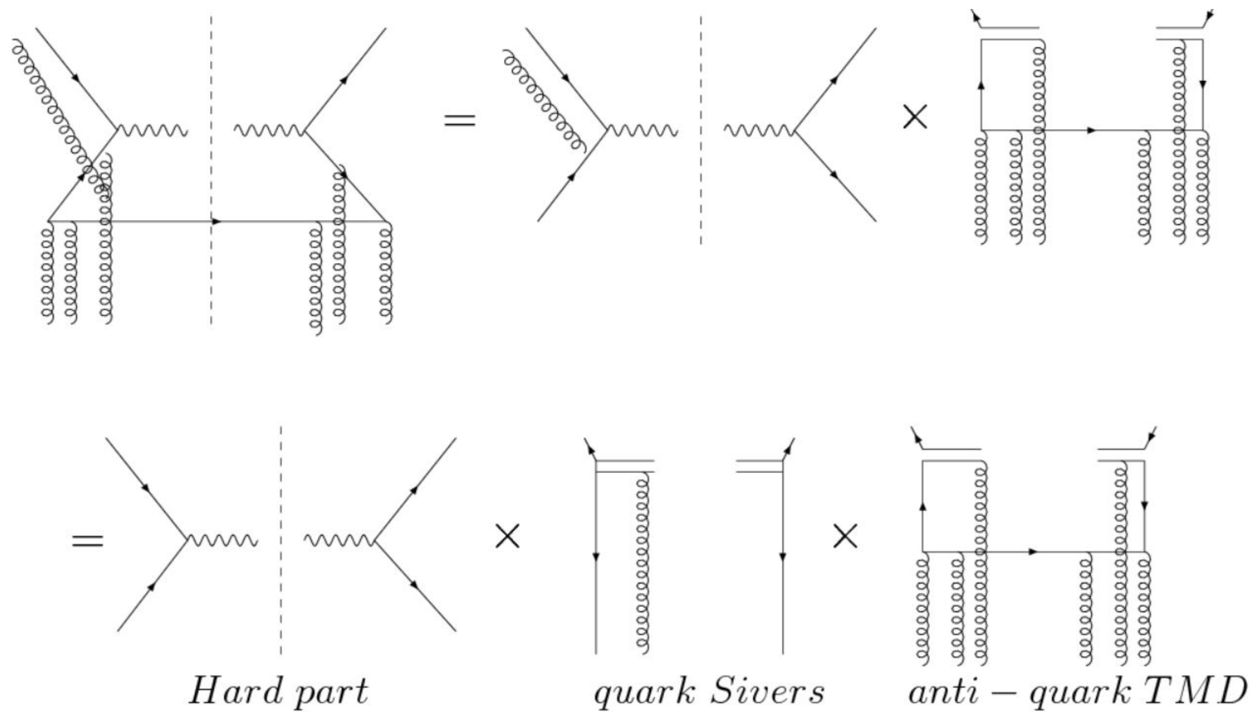
SSA at high and low Q_T

At **high Q_T**, G_4 appears, CE effect plays a role.

At **low Q_T**, the systematical cancelation occurs between SGP and HGP contributions, G_4 drops out.

Schematically,

low Q_T



At low Q_T, our result reduces to that from the standard TMD&CGC by

Z. B. Kang and B. W. Xiao

The SSA for photon-jet production in pA collisions

Disclaimer: Our treatment is less solid in this case.

In the Generalized TMD(without CE effect):

$$
\begin{aligned}
\frac{d\sigma}{dP.S.} &= \sum_q H_{Born} \int d^2k_{1\perp} d^2k_{2\perp} \delta^2(q_\perp - k_{1\perp} - k_{2\perp}) \\
&\times \Big\{ x f_q(x, k_{2\perp})\, x'_g G_{DP}(x'_g, k_{1\perp}) \\
&+ \frac{\varepsilon_\perp^{ij} S_\perp^i k_{2\perp}^j}{M_N} x f_{1T,q}^{\perp, qg\to\gamma q}(x, k_{2\perp}) x'_g G_{DP}(x'_g, k_{1\perp}) \Big\}
\end{aligned}
$$

A. Bacchetta, C. Bomhof, U. D' Alesio, P. J. Mulders and F. Murgia, 2007

- Key observation: $k_{1T} >> k_{2T}$ in the forward region of the polarized proton.

1: integrating out k_{1T}, 2: making power expansion k_{2T}/q_T,
3: using the identity:

$$
\frac{N_c^2+1}{N_c^2-1} T_{F,q}(x,x) = \int d^2k_{2\perp} \frac{k_{2\perp}^2}{M_N} f_{1T,q}^{\perp, qg\to\gamma q}(x, k_{2\perp})
$$

D. Boer, P.J. Mulders and F. Pijlman, 2003

We arrive at,

$$\frac{d\sigma}{dP.S.} \approx \sum_q H_{Born} \Big\{ x f_q(x)\, x_g' G_{DP}(x_g', q_\perp) - \varepsilon_\perp^{ij} S_\perp^i q_\perp^j \frac{N_c^2+1}{N_c^2-1} x T_{F,q}(x,x) \frac{\partial}{\partial q_\perp^2} x_g' G_{DP}(x_g', q_\perp) \Big\}$$

- Single gluon exchange from proton contributes to the k_T^2 moment of the Sivers function.
- Multiple gluon exchange contribute to k_T^4 , k_T^6 ... moment of the Sivers function and suppressed in the power of Λ_{QCD}/Q_s in the semi-hard region.

Taking into account CE effect:

one gluon exchange from proton, gluon rescattering summed to all orders on nucleus side,

$$\frac{d\sigma^{p^\uparrow A\to\gamma q+X}}{dP.S.} \approx \sum_q H_{Born}\Big\{x f_q(x)\, x'_g G_{DP}(x'_g, q_\perp)$$
$$-\varepsilon_\perp^{ij} S_\perp^i q_\perp^j x T_{F,q}(x,x)\frac{N_c^2+1}{N_c^2-1}\frac{\partial}{\partial q_\perp^2} x'_g G_{DP}(x'_g, q_\perp)$$
$$+\,\varepsilon_\perp^{ij} S_\perp^i q_\perp^j x T_{F,q}(x,x)\frac{2N_c^2}{N_c^2-1}\frac{\partial}{\partial q_\perp^2} x'_g G_4(x'_g, q_\perp)\Big\}$$

- Polarized cross section: CE effect is $1/N_c^2$ suppressed,
- Unpolarized cross section: not affected by CE effect.

$$G_4 = \frac{1}{N_C^2} G_{DP}$$

Summary

- At high transverse mometumn: CE effect plays a role
- At low transverse momentum:

  1: CE effect is absent when color flow is simple;

  2: CE effect appears when color flow is complicated.
- The SSA for photon-jet production

  1: the predictive power of GTMD is partly recovered in the forward semi-hard region,

  2: it remains to be a promising channel to **test sign change prediction.**